

Reconciling Flow Measurement Errors of a Gas Compression Unit

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Kuwait 4th Flow Measurement
Technology Conference
03rd-05th Dec 2019



Kuwait 4th Flow Measurement Technology Conference

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Objective

- Objective: To carry out the data reconciliation on a compressor system
- Reconciliation literally means a process through which people with different opinions agree in a friendly manner



What is Data Reconciliation (DR)

Data reconciliation (DR) is a process of rectifying erroneous data

System Constraints are satisfied

Uses mathematical modelling

Works best with Random Errors

Wonder what is data reconciliation



Data Reconciliation

- Data Reconciliation is basically a constrained optimization problem
- The objective function is optimized (error minimized)
- The constraints are satisfied ($A+B=C$)



Need for Data Reconciliation

Measured Data may contain errors

Such data is often non repeatable

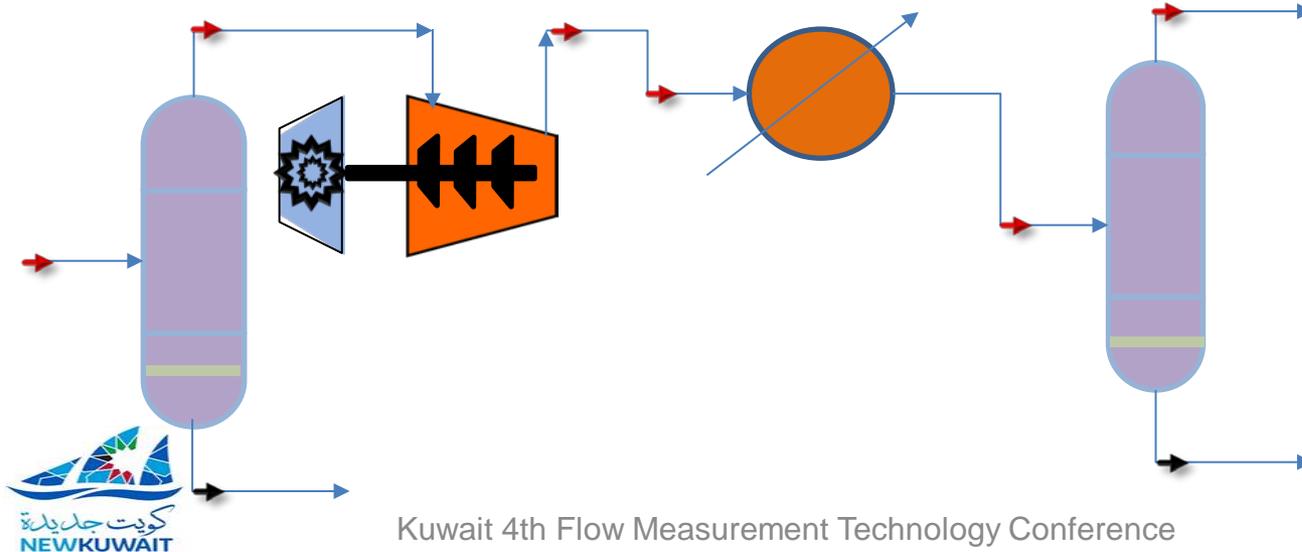
Some data may be missing or has not been measured

Data may not satisfy Process Constraints



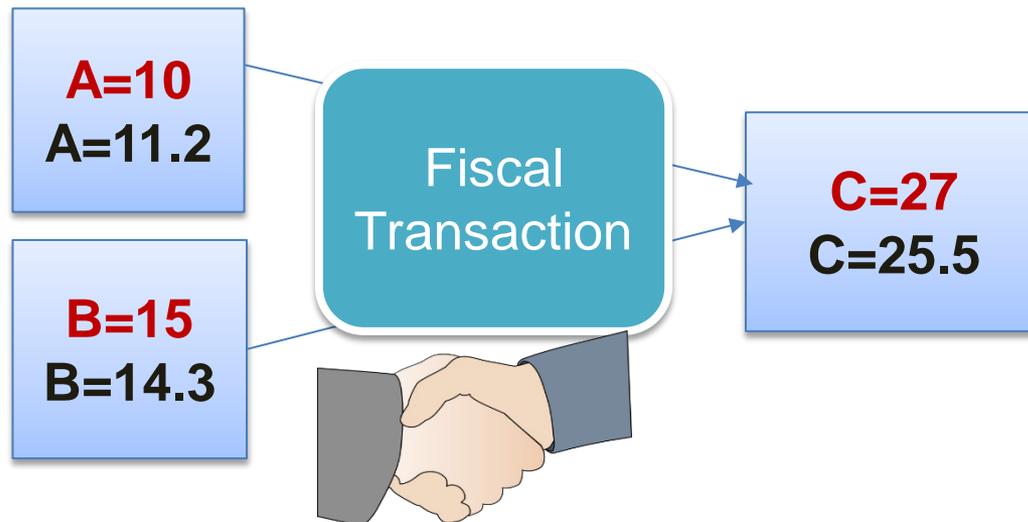
Data Reconciliation (DR) for Compressor

- Differences between true and measured values need to be minimized
- Constraints of mass and energy balances need to be fulfilled

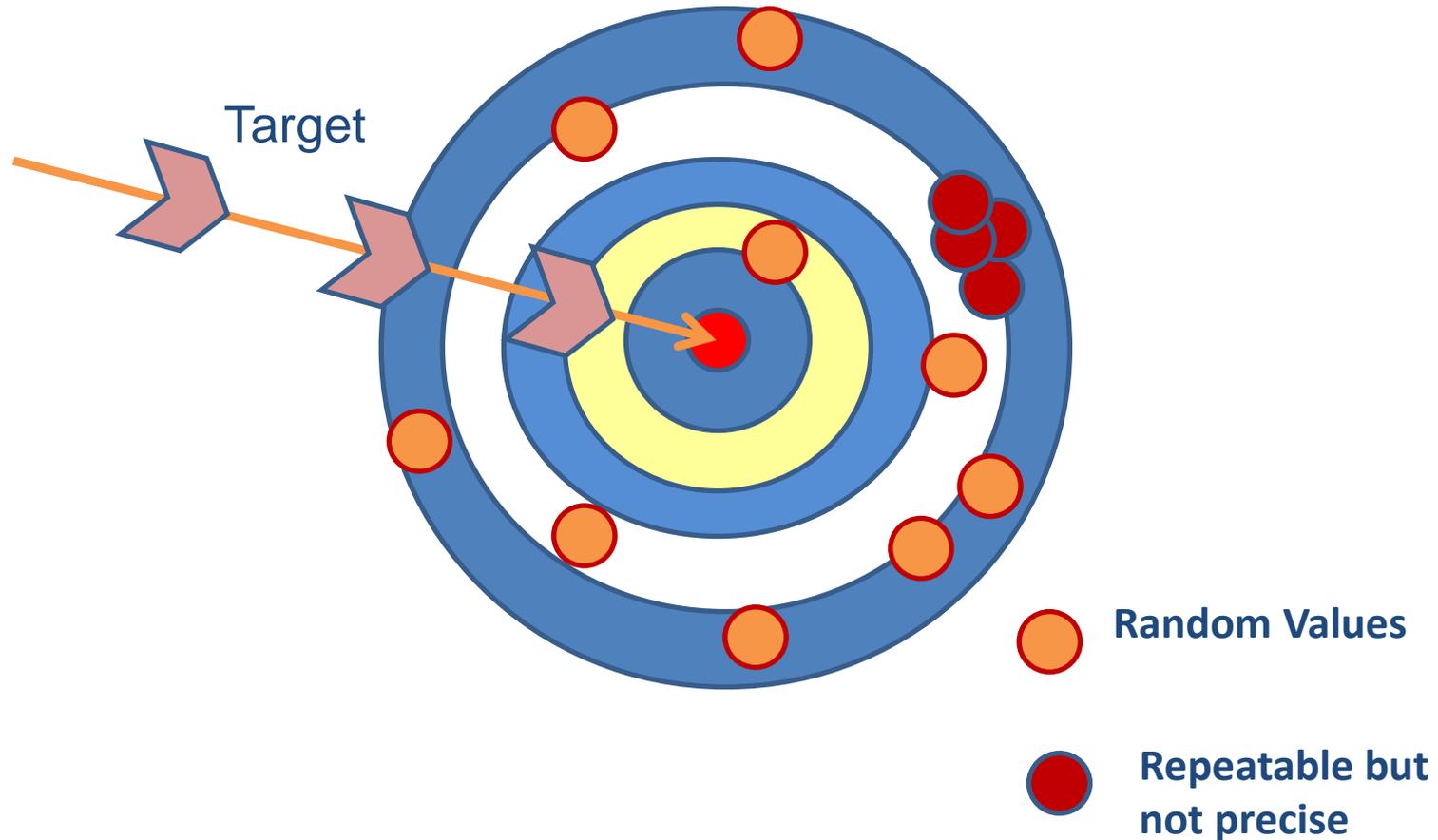


Custody Transfer

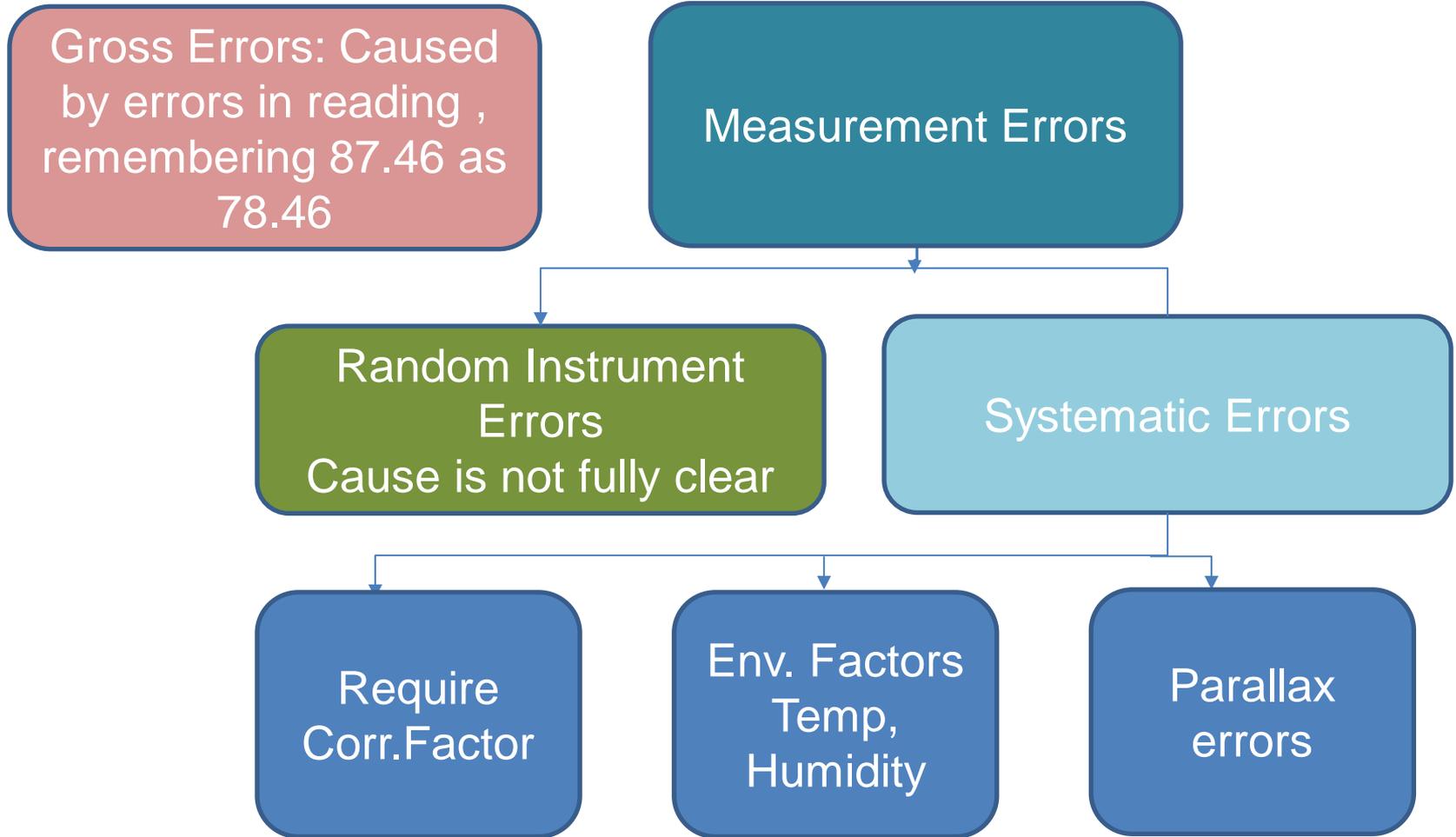
- Most critical requirement of Data Reconciliation is for custody transfer application
- It may benefit and mutually satisfy both parties involved as custody transfer often is linked to fiscal transactions



Repeatability & Accuracy



Errors in Measurement



Random Errors

Random errors are the Instrument indicated error as no Instrument is perfect

Expected to be scattered in a Normal or Gaussian Distribution

DR is best applicable to random errors

Instrument Accuracy

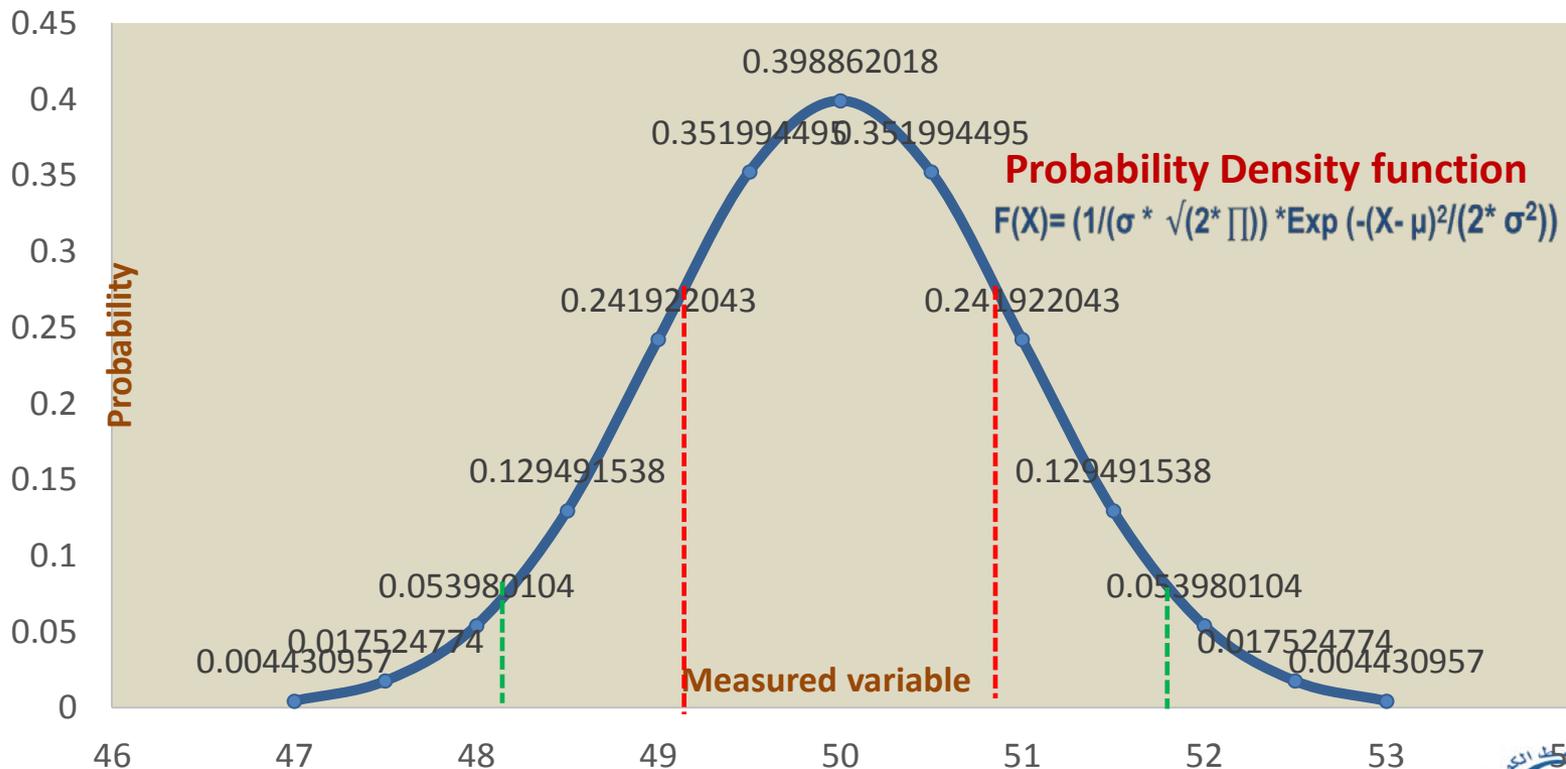
Accuracy of a meter is often defined to be $\pm 1-3\%$ (avg $\pm 2\%$) as a percentage of full scale reading with a confidence level of 95%

If the full scale reading which the Instrument can read is 1500 kg, and its error is $\pm 1\%$, then it can be said with a 95% confidence that the true value shall lie within ± 15 kgs of the measured value

The accuracy may sometimes also be specified in terms of the actual reading.

Probability Density Function-Bell Curve

- Horizontal axis is the range of possible numeric outcomes
- Vertical axis is the probability of outcome



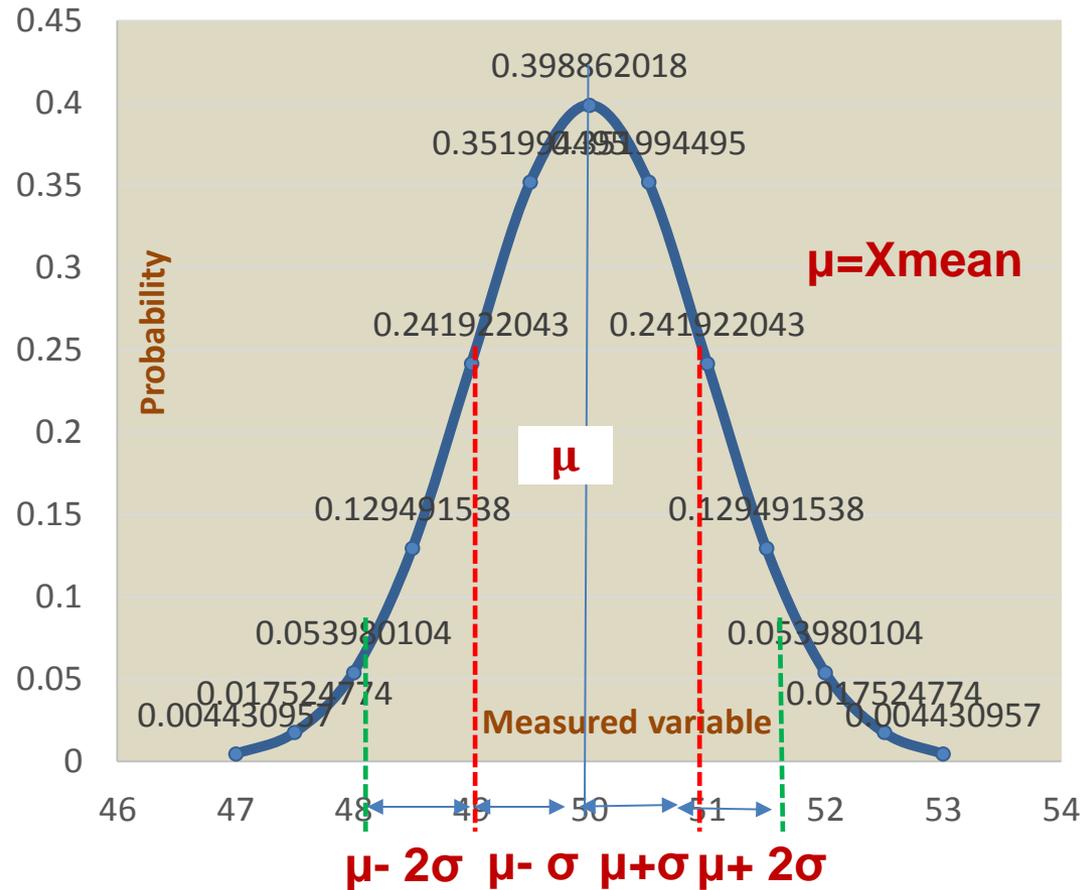
Probability Density Function & Confidence Levels

$$F(X) = (1/(\sigma * \sqrt{2*\pi})) * \text{Exp} (-(X- \mu)^2/(2* \sigma^2))$$

➤ 95% Confidence Level:
95% probability for which
 $P(X1 < X < X2) = 0.95$
X = measured value

In gaussian distribution curve

- $X1 = \mu - 1.96\sigma$ ($\sim 2\sigma$)
- $X2 = \mu + 1.96\sigma$ ($\sim 2\sigma$)



Standard Deviation

The Weight of a block is measured as follows:

S.No	Weight, in gms, Xi
1	50.8
2	51.4
3	49.6
4	48.7
5	50.2
6	49.4
7	49.9
Sum = $\sum Xi$ =	350
Mean = $\mu = \sum Xi / n =$	50
Std. Dev = $\sigma = \sqrt{\sum (Xi - \mu)^2 / (n - 1)}$	0.9000

The Probability Function Values

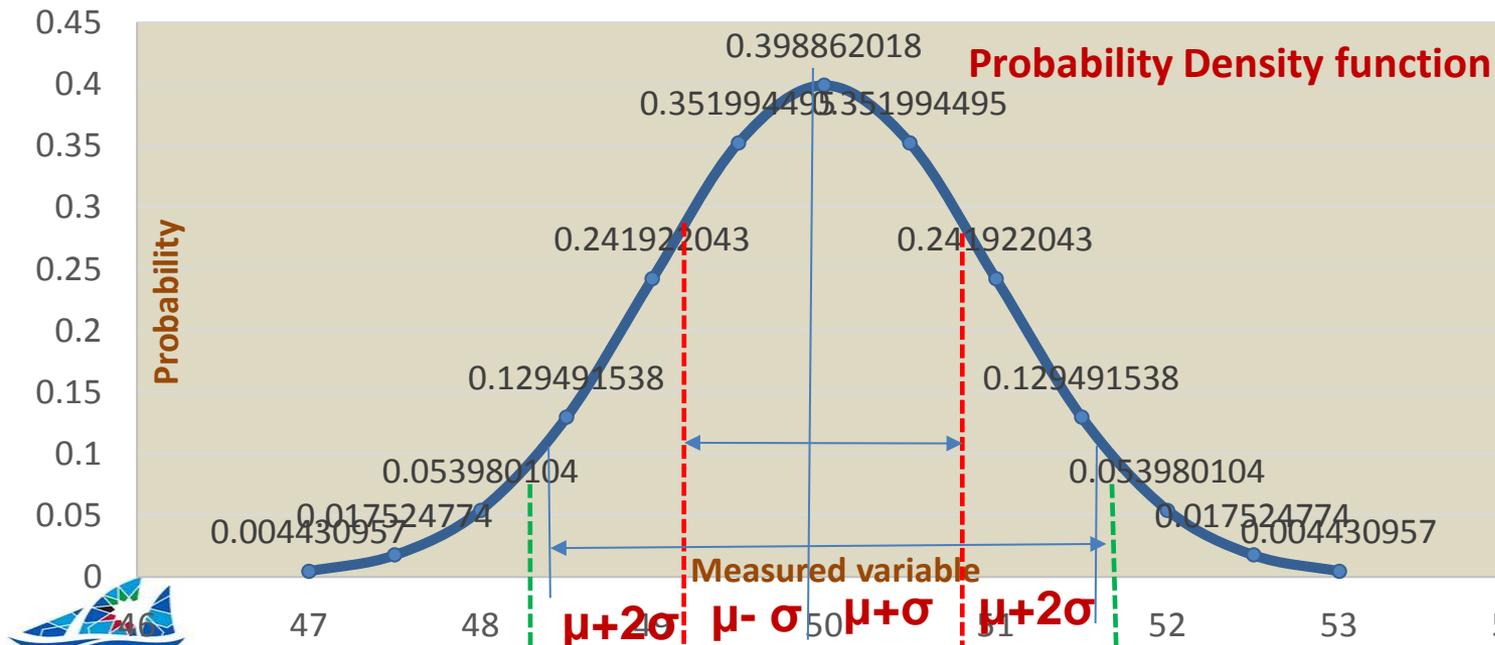
$$F(X) = (1/(\sigma * \sqrt{2*\pi})) * \text{Exp} (-(X- \mu)^2/(2* \sigma^2))$$

X	μ	σ	F(X)
48	50	0.9	0.05398
48.5	50	0.9	0.129492
49	50	0.9	0.241922
49.5	50	0.9	0.351994
50	50	0.9	0.398862
50.5	50	0.9	0.351994
51	50	0.9	0.241922
51.5	50	0.9	0.129492
52	50	0.9	0.05398

Probability of measured variables

➤ Our interest: Probability of finding measured variables in a range (between an X1 & X2)

$$P(X_1 < X < X_2) = \frac{1}{(\sigma \cdot \sqrt{2 \cdot \pi})} \int_{x_1}^{x_2} \text{Exp} \left(-\frac{(X-\mu)^2}{(2 \cdot \sigma^2)} \right)$$



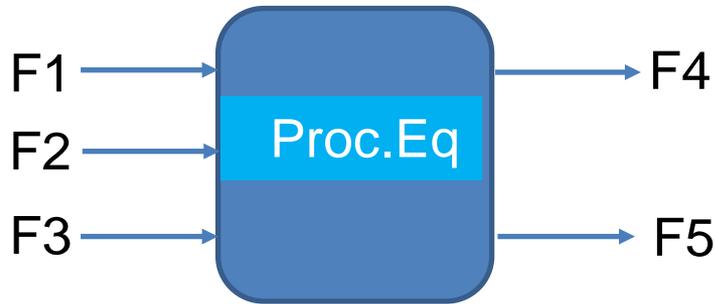
Probability-Measured values

- If Mean of X measured values $=\mu=50$, $\sigma =0.9$
- The probability that a measured value will fall between $\pm 1\sigma$ of the mean i.e between 49.1 & 50.9

$$\frac{1}{(\sigma * \sqrt{2 * \Pi})} * \int_{49.1}^{50.9} \text{Exp} (-(X-\mu)^2 / (2 * \sigma^2)) = 0.6827 = 68.27\%$$

- Likewise, Probability that the measured value will fall between $\pm 2\sigma$ i.e. between 48.2 and 51.8= **95.45%**
- Likewise, Probability that the measured value will fall between $\pm 3\sigma$ i.e. between 47.3 and 52.7= **99.7%**

Data Reconciliation – Basic Concept



Stream	Std Dev	Measured Flow, kg/Hr	Reconc . Flow
F1	5	100	98.69
F2	8	150	146.65
F3	6	350	348.12
F4	9	291	295.23
F5	10	293	298.23
Obj Fn	0.83660	Epsilon	3.6E-11

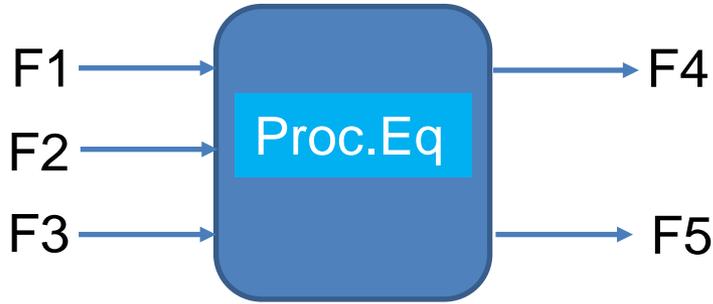
Reconc. Total=593.46

• Let reconciled flow rates = $F1^*$, $F2^*$, $F3^*$, $F4^*$ & $F5^*$

• **Minimize** $\left(\frac{(100-F1^*)}{5}\right)^2 + \left(\frac{(150-F2^*)}{8}\right)^2 + \left(\frac{(350-F3^*)}{6}\right)^2 + \left(\frac{(291-F4^*)}{9}\right)^2 + \left(\frac{(293-F5^*)}{10}\right)^2$

• **Subject to the constraints:** $((F1^*+F2^*+F3^*-F4^*-F5^*))^2 < (1 \times 10^{-6})$

Data Reconciliation –Effect of Std. Dev.



Stream	Std Dev	Measured Flow, kg/Hr	Reconc . Flow
F1	10	100	94.77
F2	9	150	145.76
F3	8	350	346.66
F4	5	291	292.31
F5	6	293	294.88
Obj Fn	0.83660	Epsilon	3.43E-10

Reconc. Total=587.19

• Let reconciled flow rates =F1*, F2*,F3*, F4*& F5*

• **Minimize** $\left(\frac{100-F1^*}{10}\right)^2 + \left(\frac{150-F2^*}{9}\right)^2 + \left(\frac{350-F3^*}{8}\right)^2 + \left(\frac{291-F4^*}{5}\right)^2 + \left(\frac{293-F5^*}{6}\right)^2$

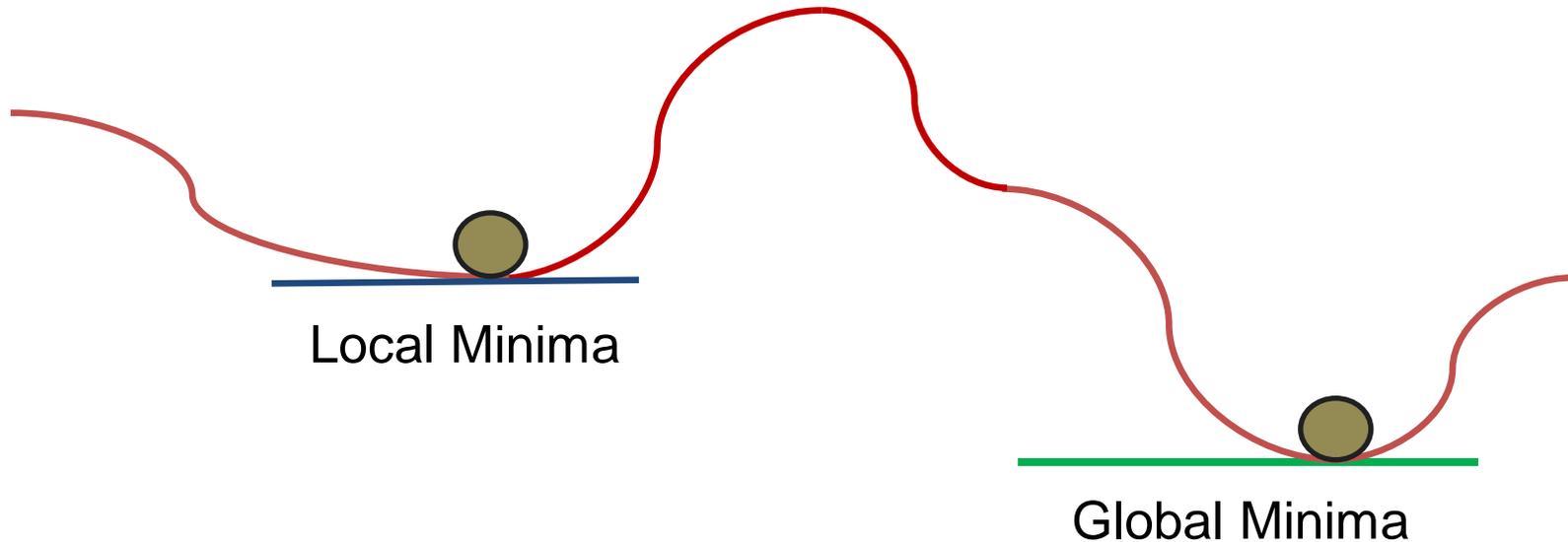
• **Subject to the constraints:** $\left((F1^*+F2^*+F3^*-F4^*-F5^*)^2\right) < (1^*e-06)$

Weighted Least squares method

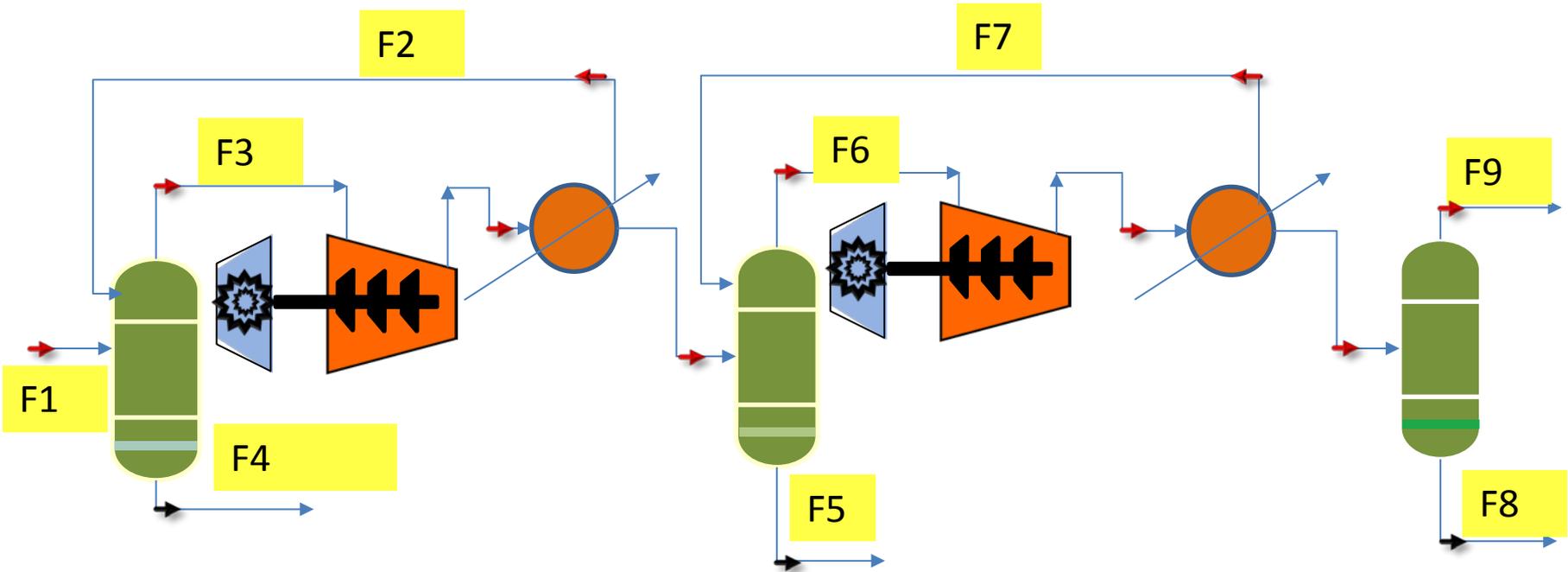
- Monitors the gradient or slope of the objective function as the input values (or decision variables) change
- an optimum solution is obtained when the partial derivatives are equal to zero.
- input values (or decision variables) change to final reconciled values
- Can get trapped into a local minima-depending on initial conditions
- Global Minima is achieved only when $\epsilon < 10^{-6}$

Data Reconciliation- Process Software Solver

- The mass and energy balances were ensured
- Achieves convergence (global minima) with any random initial conditions



Compressor System-Blind data used



Material Balance:

$$F1+F2-F3-F4=0$$

$$F3-F2+F7-F6-F5=0$$

$$F6-F7-F8-F9=0$$

Data Reconciliation-Weighted Least Squares

➤ The weights are used as the inverse of variances ($w=1/ \sigma^2$)

➤ **Minimize:** $\sum ((\text{Measured value}-\text{Reconciled Value})/ \sigma)^2$

➤ **Constraints:**

$$(F1+F2-F3-F4)^2 < \text{Epsilon (typically } 10^{-6}\text{)}$$

$$(F3-F2+F7-F6-F5)^2 < \text{Epsilon}$$

$$(F6-F7-F8-F9)^2 < \text{Epsilon}$$



Comparison of Excel Solver results vs Software

Parameters Reconciled		Excel Solver	Process Software
F1	Feed	5142	5176
F2	Recycle-1	569	569
F3	Feed to 1 st Comp	5711	5745
F4	1st Stg.Suc Scrub Liq	0	0
F5	1st Stg Disch Scrub Liq	77.3	76.1
F6	Feed to 2nd Comp	5626	5660
F7	2nd stg. Recycle	561	561
F8	2nd Stg Scrubber Liq	101	130
F9	Export Gas	4963	4970
	Constraint	<10⁻⁸	<10⁻¹⁰

Observations & Conclusions

- It can be seen that the results of weighted least squares method are based only on material balances and ignores the energy balances (i.e. purely on measured flow data)
- The Process Solver carries out the reconciliation with the both mass and energy Balances
- Though the weighted least squares method results are based only on measured flow rates, measured data is based on heat & material balance.
- Hence the difference between the weighted least squares method and the Process Software reconciliation software is not significant (within 2-3 standard deviations) of any measured variable

Thank You

