Flow Measurement Uncertainty: Standard Deviation and Standard Error of the Mean

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MEASUREMENT UNCERTAINTY

M i s c o n c e p t i o n :

- **Measurement is an exact science.**

F a c t s :

- **All measurements are merely Estimates of the True value being measured and yet the true value can never be known.**

- **The Repeated Measurement of a fixed quantity will NEVER yield the same result every time.**

MEASUREMENT UNCERTAINTY

D e f i n i t i o n :

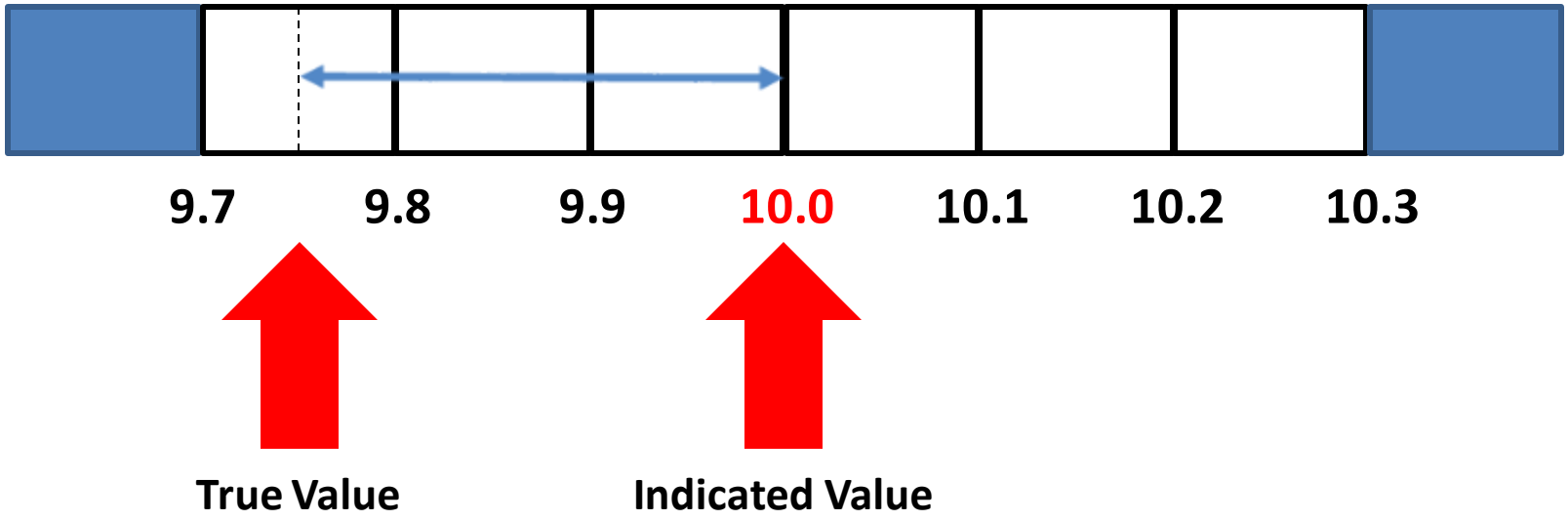
Uncertainty of measurement gives an indication of the Quality or Reliability of a measurement result.

Hence, Uncertainty is the **margin of doubt** associated with a measurement.

DON'T CONFUSE ERROR WITH UNCERTAINTY!

Error is the **difference** between the measured **value** and the **true** value.

An illustration of measurement error:



Uncertainty: Terminology

Accuracy, Precision and Error

- Accuracy describes how correct – how close to the *true answer* – the results are.
- Precision describes how repeatable they are.
- **Systematic errors** are repeated in the same way throughout an investigation, such as using a balance incorrectly in the same way for each measurement. This can be corrected. Precision describes how repeatable they are.
- **Random error** cannot easily be corrected as it affects measurements differently.

	Results A	Results B	Results C
Accuracy	Low	High	Low
Precision	High	High	Low
Systematic error	High	None	No
Random Error	No	None	High

Image source: Bishop, M. *Precision vs Accuracy (Fig 1.12).* From *Preparatory Chemistry* (ebook) via http://preparatorychemistry.com/Bishop_Book_1_eBook.pdf

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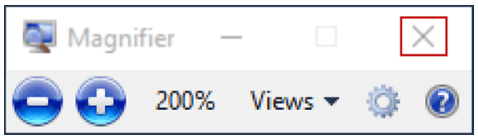

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THE EFFECT OF ERRORS

Example:

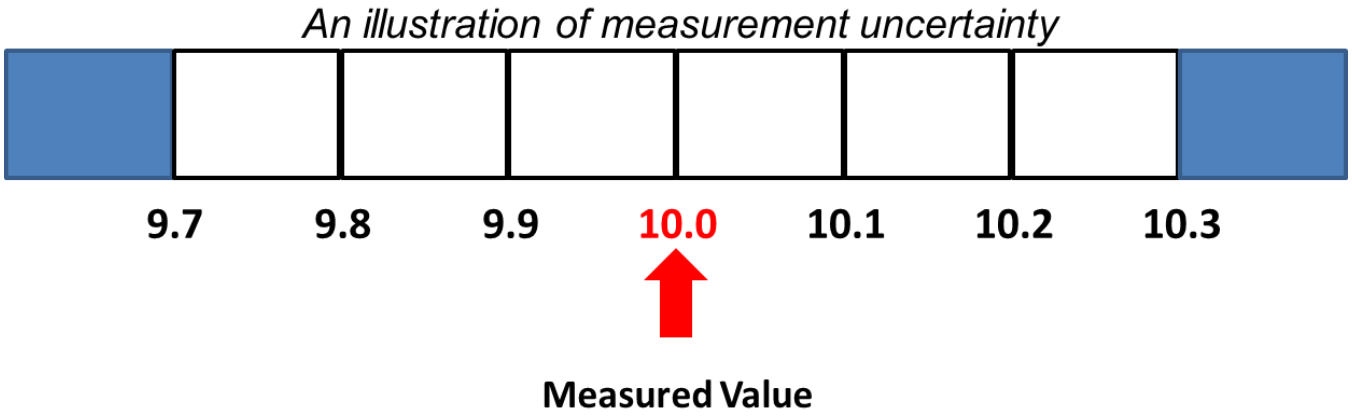
+1%

Over-reads by 1% you will lose $10,000 every day

EXPRESSING UNCERTAINTY!

This is a measure of the likelihood that the True Value of a measurement lies in the defined Uncertainty Interval.

In industry, the confidence level is usually set at **95**%.

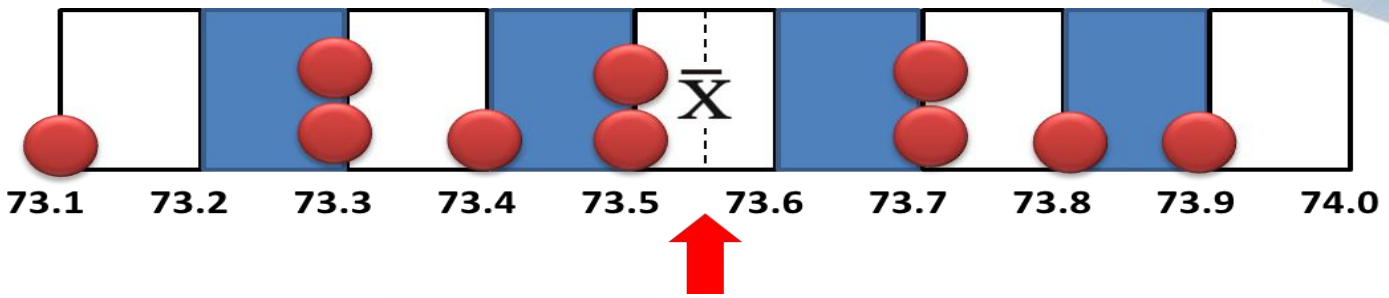


Uncertainty at 95% Confidence is 3%.

95% confident: true value of this measurement lies between **9.7** and **10.3** m^3/hr.

10.0 ± 0.3 @ **95%** confidence

Type A analysis



1. Arithmetic mean

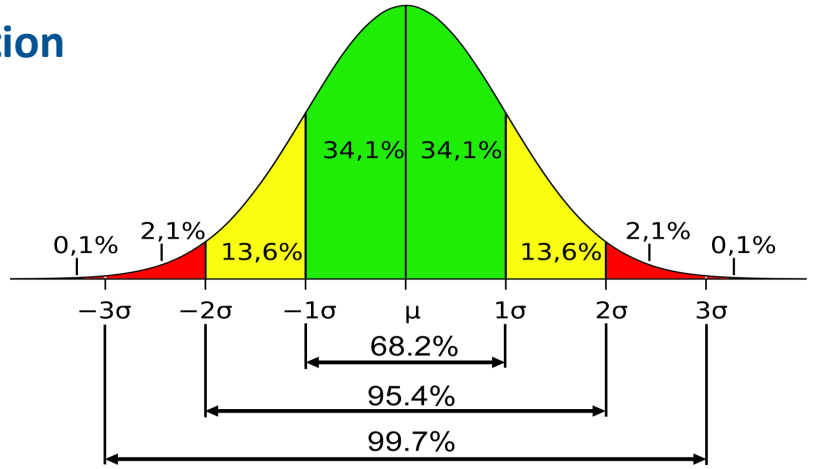
$$\overline{X} = \frac{\sum X}{N}$$

2.Spread or standard deviation

$$S = \sqrt{\frac{\Sigma(X - \overline{X})^2}{N}}$$

where S = the standard deviation of a sample,
Σ means "sum of,"
X = each value in the data set,
$\overline{X}$ = mean of all values in the data set,
N = number of values in the data set.

3. Normal or Gaussian Distribution



CALCULATION METHOD

Arithmetic mean

Suppose that a turbine meter is used to measure the flow of water from a borehole. Readings are taken every two hours in the course of a single day and are (in m^3/hr)

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
126.4	133.5	129.5	137.8	123.2	128.6	130.7	131.2	135.6	126.9	127.4	133.9

Mean: $\overline{X} = \frac{\sum X}{N}$ **= 130.4 m^3/hr**

useful to know what the spread of the measurements is.

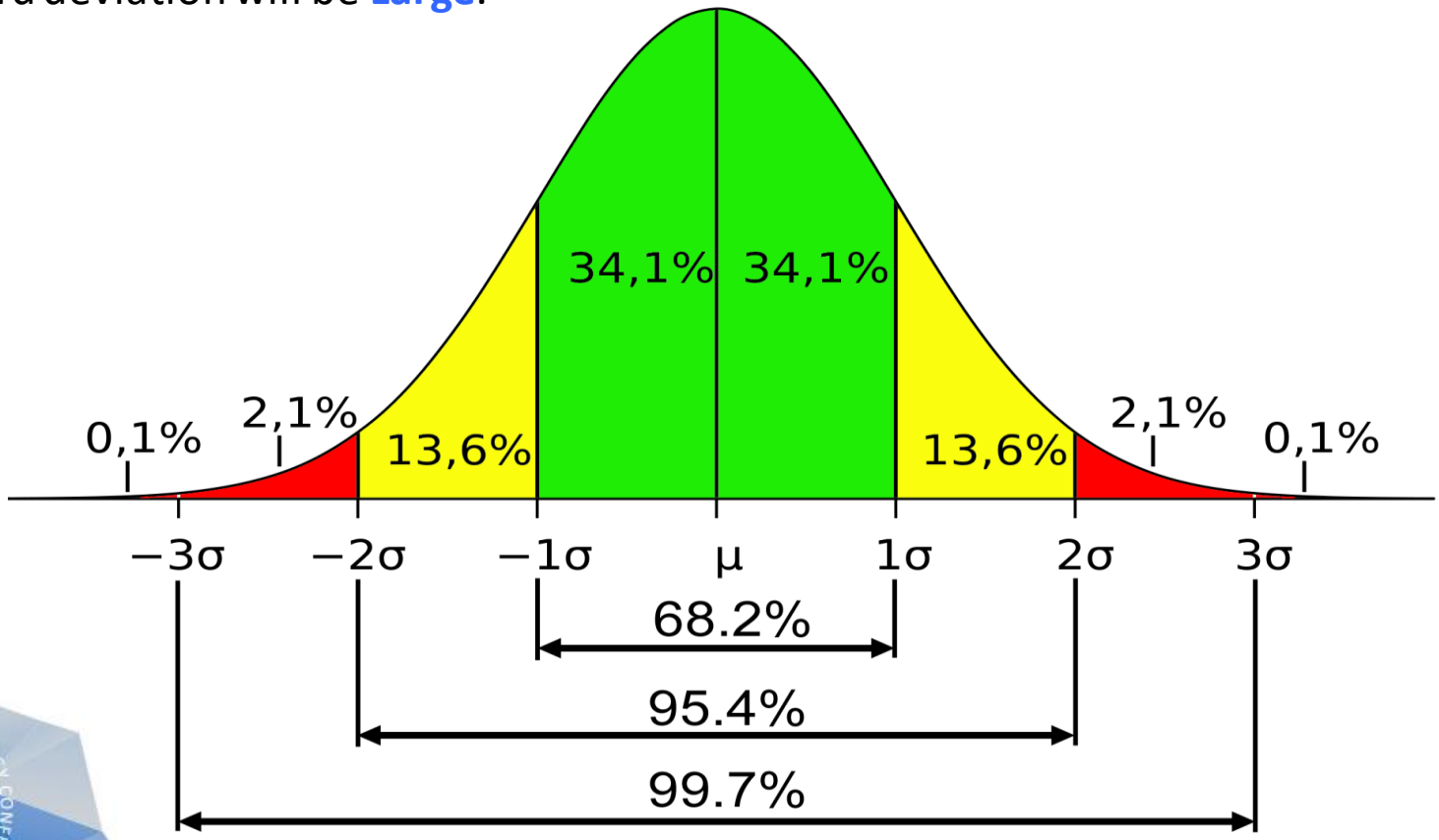
CALCULATION METHOD

2. Standard Deviation (σ)

It's a unit of measure & shows the **Variation** in **Data**.

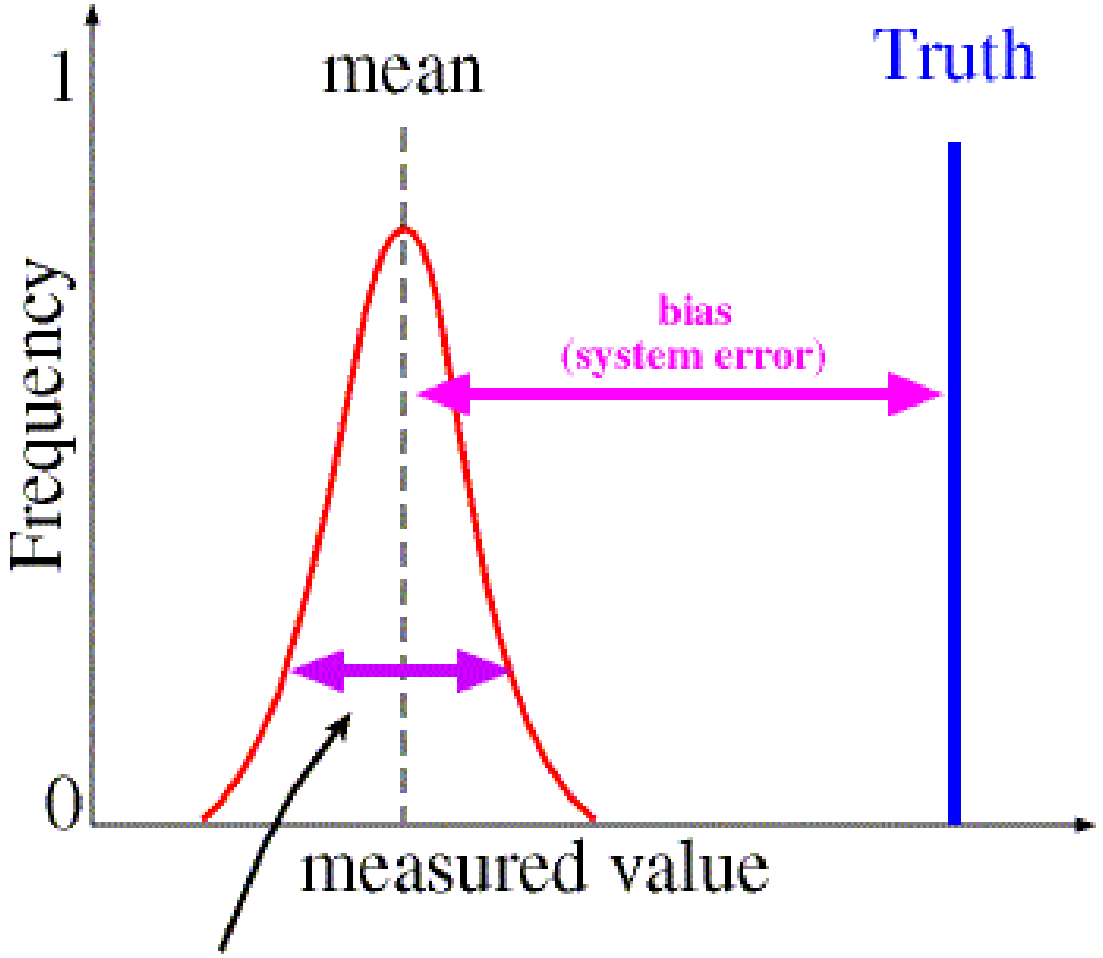
If the data is **Close** together, the standard deviation will be **Small**. If the data is spread out,

the standard deviation will be **Large**.



CALCULATION METHOD

S t a n d a r d D e v i a t i o n



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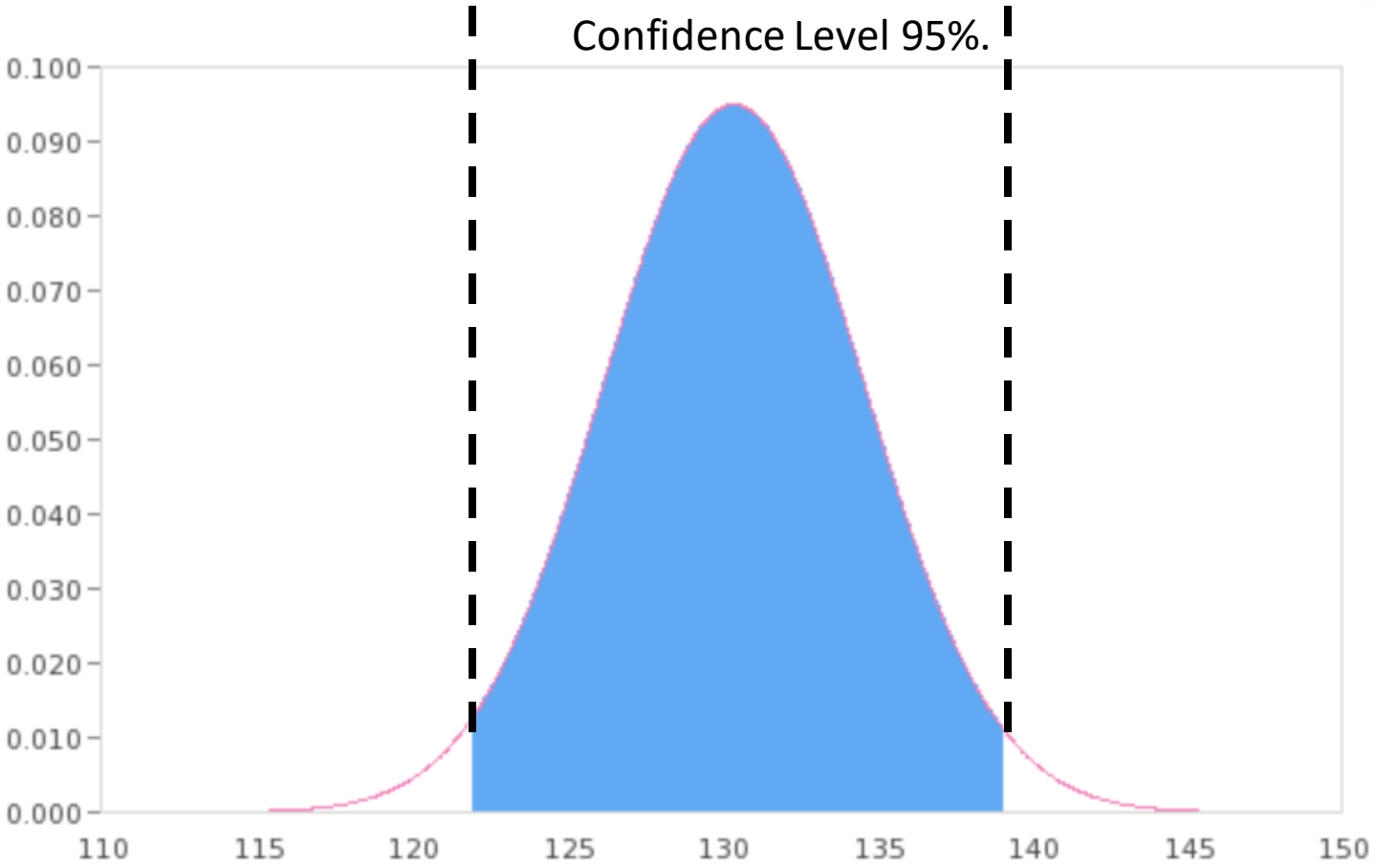
STDEV: $\sigma = \sqrt{\dfrac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$ **= 4.2 m^3/hr**

Mean: 130.4 ± 4.2 m^3/hr

CALCULATION METHOD

3. Normal Distribution

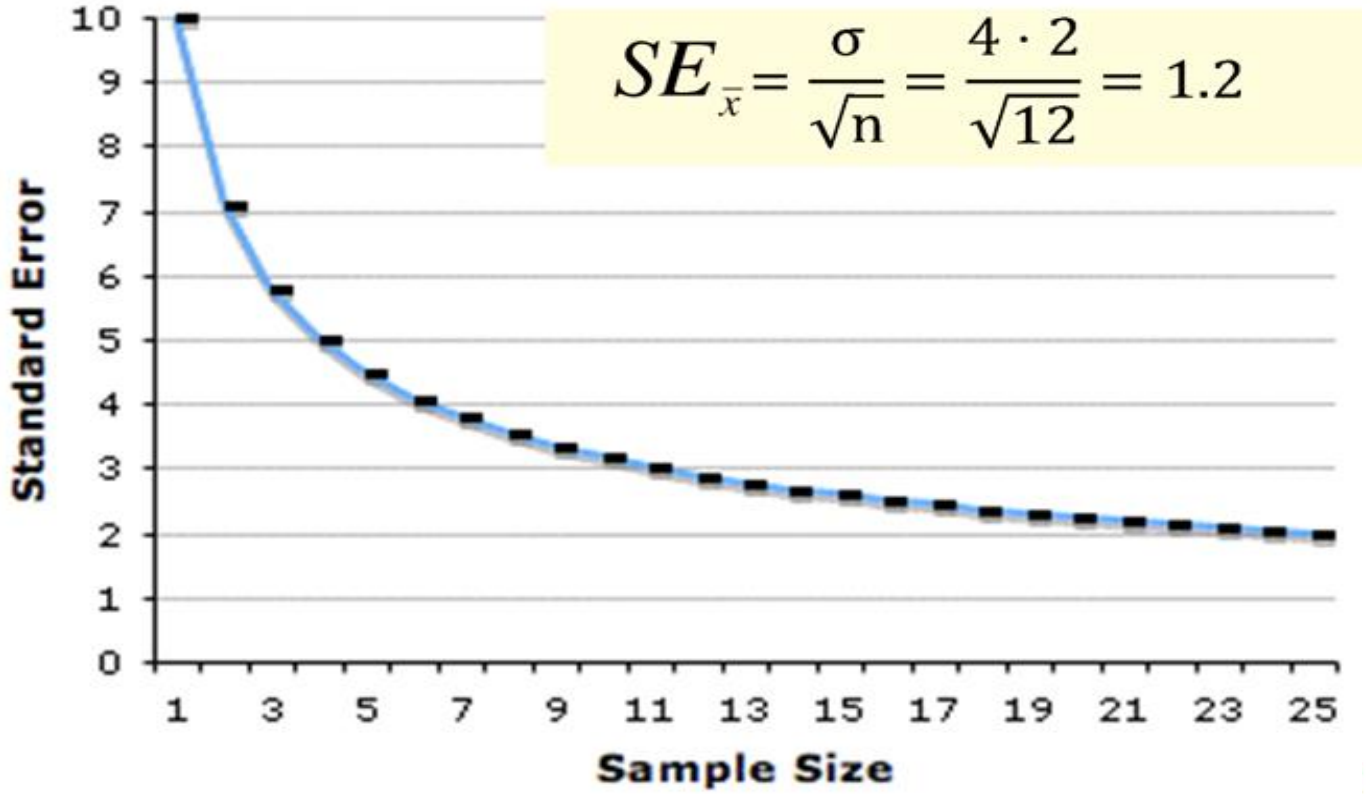


Standard Error of the Mean

Difference between **SEM** & **SD**:

- Standard deviation - it is a measure of data dispersion.
- Standard Error of the mean - refers to the probability that sample means, not individual data points, differ from the true population mean

$$SE_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{4 \cdot 2}{\sqrt{12}} = 1.2$$



Conclusion

Standard Deviation and Standard Error of the Mean

However, Often it is impossible to assess the magnitude of the uncertainty from repeated measurements and have to be quantified using other means.

For example, these could be:

- The uncertainty quoted on a calibration certificate
- Engineering judgement based on experience of a measurement system
- Manufacturer's specifications

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Questions & Discussions

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Thank you all for your time & attention

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